

Additional ways to determination of structure of high energy elastic scattering amplitude

O. V. Selyugin¹ *

Bogoliubov Laboratory of Theoretical Physics,

Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

Abstract

Several methods of extracting the magnitude of the real parts of the elastic hadron scattering amplitude - from the experimental data are presented. These methods allows us to obtain the real parts at one definite point of the transfer momentum and with a high accuracy without knowledge of the precise values of the normalization coefficient and with minimum theoretical assumptions.

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*selugin@thsun1.jinr.ru

The obtaining of the structure of the hadron scattering amplitude is an important task both for a theory and an experiment. QCD has a certain difficulty in calculating the magnitude of the scattering amplitude in the diffraction range, as well as its phase and the energy dependence in the diffraction range. For our deeper understanding, the work of such a fundamental relation as dispersion and local dispersion relations requires the knowledge of the structure of the scattering amplitude with an accuracy, we can obtain [1]. Also note that in [2] it was shown that the knowledge of the behavior of ρ - the ratio of the real to imaginary part of the spin-non-flip amplitude can be used for checking the local QFT already in the LHC energy region.

A large number of experimental and theoretical studies of the high energy elastic proton-proton and proton-antiproton scattering at small angles gives a rich information about this process, which allows to narrow the circle of examined models and to put a number of the difficult problems which are not solved entirely in the meanwhile. Especially this concerns the energy dependence of a number of characteristics of these reactions and the contribution as the odderon.

A great deal of these questions are connected with the dependence of the spin-non-flip phase of hadron-hadron scattering with s and t . The most of the models define the real part of the scattering amplitude phenomenologically. Some models used the local dispersion relations and the hypothesis of the geometrical scaling. As is known, using some simplifying assumption, the information about the phase of the scattering amplitude can be obtained from the experimental data at small momentum transfers where the interference of the electromagnetic and hadron amplitudes takes place. On the whole, the obtained information confirms the local dispersion relations. It was shown in [3] that the uncontradicted description of the experimental data in the range of ISR and SPS can be obtained in the case of a rapidly changing phase when

the real part of the scattering amplitude grows quickly in the range of small t and becomes dominant.

The standard procedure to extract the magnitude of the real part includes the fit of the experimental data taking the magnitude of total cross section, slope, ρ , and, sometimes the normalization coefficient as free parameters:

$$\sum_i^k \frac{(nd\sigma_i/dt_{exp} - d\sigma_i/dt)^2}{\Delta_{exp}^2} \quad (1)$$

where $d\sigma_i/dt_{exp}$ is the differential cross sections at point t_i with the statistical error Δ_i extracted from the measured dN/dt using, for example, the magnitude of luminosity. The procedure require a sufficiently wide interval of t and large number of experimental points.

The theoretical representation of the differential cross-sections is

$$\frac{d\sigma}{dt} = 2\pi[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2] , \quad (2)$$

The total helicity amplitudes can be written as a sum of nuclear $\Phi_i^h(s, t)$ and electromagnetic $\Phi_i^e(s, t)$ amplitudes :

$$\Phi_i(s, t) = \Phi_i^h(s, t) + e^{i\alpha\varphi}\Phi_i^e(s, t) . \quad (3)$$

where $\Phi_i^e(s, t)$ are the leading-terms at high energies of the one-photon amplitudes as defined, for example, in [4] and the common phase φ , is

$$\varphi = -[\gamma + \log(B(s, t)|t|/2) + \nu_1 + \nu_2] \quad (4)$$

where $B(s, t)$ is the slope of the nuclear amplitude, and ν_1 and ν_2 are small correcting terms define the behavior of the Coulomb-hadron phase at small momentum transfers (see, [5]). At very small t and fixed s , these electromagnetic amplitudes are such that $\Phi_1^e(s, t) = \Phi_3^e(s, t) \sim \alpha/t$, $\Phi_2^e(s, t) = -\Phi_4^e(s, t) \sim \alpha \cdot \text{const.}$, $\Phi_5^e(s, t) \sim -\alpha/\sqrt{|t|}$. We assume, as usual, that at high energies and small angles the double-flip amplitudes are small with re-

spect to the spin-nonflip one and that spin-nonflip amplitudes are approximately equal. Consequently, the observables are determined by two amplitudes: $F(s, t) = \Phi_1(s, t) + \Phi_3(s, t)$ and $F_{sf}(s, t) = \Phi_5(s, t)$. So,

$$\begin{aligned} d\sigma/dt = \pi[(F_C(t))^2 + (ReF_N(s, t))^2 + (ImF_N(s, t))^2 \\ + 2(\rho(s, t) + \alpha\varphi(t))F_C(t)ImF_N(s, t)]. \end{aligned} \quad (5)$$

$F_C(t) = \mp 2\alpha G^2/|t|$ is the Coulomb amplitude; α is the fine-structure constant and $G^2(t)$ is the proton electromagnetic form factor squared; $Re F_N(s, t)$ and $Im F_N(s, t)$ are the real and imaginary parts of the nuclear amplitude; $\rho(s, t) = Re F_N(s, t)/Im F_C(s, t)$. Just this formula is used for the fit of experimental data determining the Coulomb and hadron amplitudes and the Coulomb-hadron phase to obtain the value of $\rho(s, t)$.

Numerous discussions of the value of $\rho(s, t)$ measured by the UA4 [6] and UA4/2 [7] Collaborations at $\sqrt{s} = 541$ GeV have revealed the ambiguity in the definition of this parameter [8], and, as a result, it has been shown that one has some trouble in extracting, from experiment, the total cross sections and the value of the real parts of the scattering amplitudes [9]. In fact, the problem is that we have at our disposal only one observable $d\sigma/dt$ for two unknowns the real part and imaginary part of the hadron spin-non-flip amplitude. So, we need either some additional experimental information which would allow us to determine independently the real and imaginary parts of the nonflip hadron elastic scattering amplitude or develop some new ways to determine the magnitude of the phase of scattering amplitude with minimum theoretical assumptions. One of the most important points in the definition of the real part of scattering amplitude is the knowledge of the normalization coefficient and the magnitude of $\sigma_{tot}(s)$.

To obtain the magnitude of $ReF_N(s, t)$, we fit the differential cross sections either taking into account the value of σ_{tot} from another experiment, to

decrease the errors, as made by the UA4/2 Collaboration, or taking σ_{tot} as a free parameter, as made in [8]. If one does not take the normalization coefficient as a free parameter in the fitting procedure, its definition requires the knowledge of the behavior of imaginary and real parts of the scattering amplitude in the range of small transfer momenta and the magnitude of $\sigma_{tot}(s)$ and $\rho(s, t)$.

Note three points. First, in any case, we should take into account the errors in $\sigma_{tot}(s)$. Second, this method means that the imaginary part slope of the scattering amplitude equals the slope of its real part in the examined range of transfer momenta, and for the best fit, one should take the interval of transfer momenta sufficiently large. Third, the magnitude of $\rho(s, t)$ thus obtained corresponds to the whole interval of transfer momenta.

In this report, we briefly describe some new procedures of simplifying the determination of elastic scattering amplitude parameters.

From equation (5) one can obtain the equation for $ReF_N(s, t)$ for every experimental point - i

$$\begin{aligned} ReF_N(s, t_i) &= -ReF_C(s, t) \\ \pm [n/\pi d\sigma/dt(t = t_i) - (ImF_C(t_i) + ImF_N(t_i))^2]^{1/2}. \end{aligned} \quad (6)$$

As the imaginary part of scattering amplitude is defined by

$$ImF_N(s, t) = \sigma_{tot}/(0.389 \cdot 4\pi) \exp(B/2t), \quad (7)$$

it is evident from (6) that the determination of the real part depends on n, σ_{tot}, B . The magnitude of σ_{tot} determined from experimental data depends on the normalization parameter n which reflects the experimental error in determining $d\sigma/dt$ from dN/dt .

Let us examine this expression for the pp -scattering. For this aim, let us make a gedanken experiment and calculate $d\sigma/dt$ with definite parameters

taking them as the experimental points. In this case, we know exactly what we obtain at the end of our calculation. In this report we drop the full analysis of this model experiment and dwell only on the special point.

For the pp -scattering at high energies, the equation (6) has a remarkable property. If we expand the expression under the radical sign, we obtain

$$(n-1)(ImF_C + ImF_N)^2 + n(ReF_C + ReF_N)^2. \quad (8)$$

As the real part of the Coulomb scattering amplitude is negative and the real part of the nucleon scattering amplitude is positive, it is clear that this expression - Del has a minimum situated on the scale of t independent of n and σ_{tot} shown in Fig. 1 (a and b).

So, the position of the minimum gives us t_{er} where $ReF_N = -ReF_C$. As we know the Coulomb amplitude, we estimate the real part of the pp -scattering amplitude at this point. Note that all other methods give us the real part only in a sufficiently wide interval of the transfer momenta. This method works only in the case of the positive real part of the nucleon amplitude, and it is especially good in the case of large ρ . So, it is interesting for the future experiment on RHIC.

Though in the range of ISR we have small $\rho(s, t \approx 0)$ and few experimental points, let us try to examine one experiment, for example, at $\sqrt{s} = 52.8 \text{ GeV}$. This analysis is shown in Fig. 2. One can see that in this case the minimum is sufficiently large, and $-t_{min} = (3.3 \pm 0.1)10^{-2} \text{ GeV}^2$. The corresponding real part equals $0.442 \pm 0.014 \text{ GeV}$. Note that this magnitude is absolute and independent of the normalization coefficient and σ_{tot} . If we take, as in the experiment, $\sigma_{tot} = 42.38 \text{ mb}$, we obtain $\rho = 0.063 \pm 0.003$. Paper [10] gives $\rho = 0.077$.

For the RHIC energies we can make a special random procedure with the gaussian distribution of statistics and obtain, for example, the pictures

for $\rho(t) = 0.135$ and for $\rho(t) = 0.175$ shown in Fig. 3. The difference between these two representations is obvious. There is another interesting characteristic, the magnitude of second maximum. It is easy to connect the size of the maximum with the magnitude of the real part of the scattering amplitude. After differentiated the representation of Del we can determine the position of maximum of $Del = \Delta_{max}$ and obtain:

$$ReF_N(s, t) \approx \Delta_{max}(1 + \sqrt{\alpha B} \Delta_{max}^{-3/2}), \quad (9)$$

where $B/2$ is the slope of $ReF_N(s, t)$. It is to be noted that this representation slightly depends of our supposition on the form of $ReF_N(s, t)$.

From Fig.3 we can look for the ratio of these two magnitudes of $ReF_N(s, t)$. $ReF_N(s, t)_1 = 1.35$ and $ReF_N(s, t)_2 = 2.2$. These values give $\rho_1 = 0.13$ (input $\rho_1 = 0.135$) and $\rho_2 = 0.18$ (input $\rho_2 = 0.175$).

$$ReF_N(s, t)_1 / ReF_N(s, t)_2 = 0.72$$

and it can be compared with the input ratio of ρ :

$$\rho_1 / \rho_2 = .135 / .175 = 0.77$$

So, the magnitude of second maximum can give further information on the size of the real part of scattering amplitude.

This point of t_{er} is very important for the determination of the real part of spin-flip amplitude also. In the standard theory, the spin-flip hadronic amplitudes are expected to fall as $1/\sqrt{s}$ as $s \rightarrow \infty$, and they give negligible contributions at the present high energies. Only the spin-non-flip hadronic amplitudes $\Phi_1^N(s, t) \simeq \Phi_3^N(s, t)$ survive at these energies. So, the spin effects arise mostly from the interference between the non-flip hadron amplitudes with the spin-flip electromagnetic amplitudes. But there are some models which predict non-dying spin effects in the diffraction range at high energies [11,12]. The analyzing power is

$$A_N = 4\pi \text{Im}[(\Phi_1 + \Phi_3)^* \Phi_5] / d\sigma/dt, \quad (10)$$

or, separating the electromagnetic-hadron interference and pure hadron parts, we obtain

$$\begin{aligned} -d\sigma/dt A_N / (4\pi) &= \text{Im} F_N \text{Re} \Phi_5^e + \text{Im} F_N \text{Re} \Phi_5^h \\ &\quad - (\text{Re} F_C + \text{Re} F_N) \text{Im} \Phi_5^h. \end{aligned} \quad (11)$$

At the point t_{er} , the real part of the hadron spin-flip amplitude is

$$\text{Re} \Phi_5^h = -A_N (d\sigma/dt) / (4\pi \text{Im} F_N) - \Phi_5^e. \quad (12)$$

So, measuring the analyzing power at this point, we can obtain the magnitude of the real part of the hadron spin-flip amplitude without any theoretical assumption about its t -dependence.

Note, that $A_N d\sigma/dt = \sigma(++) - \sigma(+-)$. Hence the determination of the magnitude of the real part of hadron spin-non-flip amplitude contains only the difference of the cross sections with the parallel and antiparallel spins of nucleons.

An the end, let us note some additional method which can give the independent information on the real part of the hadron spin-non-flip amplitude using the measure of A_N if the hadron spin-flip amplitude disapper at high energies [13]. Namely, from eqs. (6) and (10), we get the real part of the hadron amplitude

$$\begin{aligned} \text{Re} F_N(s, t_i) &= -\text{Re} F_C(s, t_i) \\ &\quad + [n/\pi d\sigma/dt - (\text{Im} F_C(t_i) + \tilde{A}_N n d\sigma/dt)^2]^{1/2}, \end{aligned} \quad (13)$$

where

$$\tilde{A}_N \equiv -A_N^{exp}(s, t_i) / (4\pi \cdot \Phi_5^e(t_i)). \quad (14)$$

Again the real part of scattering amplitude is expressed in terms of the experimental quantities $d\sigma^{exp}/dt$ and A_N^{exp} , the normalisation factor n , and also in terms of the theoretically known electromagnetic amplitude Φ_1^e .

The precise experimental measurements of dN/dt and A_N at RHIC, as well as, if possible, at the Tevatron, will therefore give us unavailable information on the hadron elastic scattering at small t . New phenomena at high energies could be therefore detected without going through the usual arbitrary assumptions (such as the exponential form) concerning the hadron elastic scattering amplitudes.

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Captions

FIG.1. The model calculations of the del for the pp -scattering at RHIC energy $\sqrt{s} = 540 GeV$ on different n

a) with $\sigma_{tot} = 63 mb$; b) with $\sigma_{tot} = 62 mb$.

FIG.2. The calculation of Del for the pp -scattering using the experimental data at $\sqrt{s} = 52.8 GeV$. The lines are the polynomial fit of the points calculated with experimental data and with different n

FIG.3. The calculation of del for the model pp -scattering with a) $\rho_1 = 0.135$ and b) $\rho_2 = 0.175$. The solid, short-dashed, and dotted lines are the theoretical curves for $\rho_2 = 0.175$, $\rho_1 = 0.135$, $\rho_0 = 0$. respectively.

FIGURES







